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DATA COMMUNICATION NETWORKS AND INFORMATION THEORY *

Pierre A. Humblet

Laboratory for Information and Decision Systems
Massachusetts Institute of Technology
Cambridge, Massachusetts

ABSTRACT

Some contributions of Information Theory to the study of data communication networks are outlined, with special attention given to the problem of efficient message addressing and to the capacity of Aloha-like multiple access channels with infinite number of sources.

I. INTRODUCTION

The field of data communication networks has been one of the most active topics of research in communication systems during the past decade. Information Theory had a modest role in that study, especially when compared to its prominent influence as underlying the theories of modulation and coding. However, besides its study of multi-user source and channel coding, Information Theory did contribute to two challenging aspects of data networks, in providing bounds on the amount of information necessary to address messages, and bounds on the efficency of "Aloha-like" multiple access channels.

In their simplest forms, both problems are similar: a number of sources can transmit on a common channel. In the addressing problem, a "server" observes which sources have messages ready, and selects a message for transmission on the line. The problem is to decide which message to select, and to encode efficiently from what source the message originates. In the "Aloha-like" multiple access channel, there is no central observer. Rather sources observe the channel and decide individually when to transmit. If only one source at a time transmits, the message goes through correctly. If two or more transmit simultaneously, all messages are lost and the sources must retry later. The problem is to find an efficient access algorithm.

Those problems exist only because of the bursty nature of sources used in data networks (they produce short messages between long and variable periods of silence), and because it is required that messages be transmitted with short delays. If delays much longer than the typical time between the generation of messages could be tolerated, then almost no explicit information would be needed for the first problem, and collisions need not occur in the second, as, by the law of large numbers, transmissions could be scheduled (almost) deterministically after having buffered a large number of messages.

No theory has been found to deal with the questions of bursty sources and real time transmissions. The purpose of this paper is to review the contribution of Information Theory, leaving technicalities and most formulas to the references, and to generate attention to these questions.

II. MESSAGE ADDRESSING

A data communication network carries messages between a large number of users. We call the sequence of messages from a given source to a given destination a conversation. Naturally as messages are transmitted in the network one must send not only the body of the message, but also information about which conversation it belongs to. Giving this information is called addressing, as often the conversation is identified by the addresses of its source and destination.

One way to address a message is to prefix it with a codeword specifying its conversation, thus requiring $\log_2 N$ bits per message, where N is the number of conversations using a link. This explicit coding scheme has the merit of simplicity, but can be inefficient when messages are short and N is large. If the traffic on the link is heavy the inefficient encoding can cause very large delays which are not acceptable in data networks.

Another way of addressing is to cycle through the set of conversations using a link and, for each conversation, to transmit a message (or part of a message) if one is waiting, else a special "empty" codeword. Thus no explicit address is transmitted, but rather the position of a message in a cycle allows the determination of its conversation. This scheme works well for synchronous sources, and for bursty sources on heavily loaded channels where high delays guarantee that "empty" codewords are rarely sent. However it causes relatively high delays when traffic is light, as a message must wait for its turn in the cycle, thus typically $\frac{N}{2}$ times the time to send the "empty" codeword.

It is thus apparent that the addressing scheme

minimizing the expected message delay depends on the characteristics of the traffic. In light traffic the expected number of messages waiting is small compared to N, and explicit coding schemes are preferred as the conversation to which a message belongs can hardly be predicted. In heavy traffic however the number of waiting messages is large, thus one can gain by reordering the messages, so that the conversation to which the next one belongs is more predictable and requires less information. In between, one can use mixed strategies, where one cycles between groups of conversations, distinguishing between conversations in the same group by explicit addresses. The problem is complicated because the addressing scheme itself influences the delay, and thus the number of waiting messages. These concepts are developed in [1], where several schemes are analyzed and the resulting queueing delays compared; differences between schemes can be impressive in heavy traffic.

The above discussion, while not rigorous, has the merit to point out that the amount of addressing information (in the information theoretic sense) depends on the delay. This was recognized by Gallager who succeeded in quantifying it [2]. He noticed that the arrival times of the messages at the receiver give information about the times the messages were generated by the sources. That the receiver is not interested in those times does not alter the fact that the information is transmitted and uses channel capacity. The smaller the delay, the more precisely the arrival time at the receiver specifies the generation time, thus the more information is transmitted.

Gallager, using rate distortion theory with the delay as distortion, showed that if the message generations for a conversation are Poisson with rate λ, and if the expected delay before delivery at the receiver is d, then the information supplied to the receiver about message generation times is at least $\log_2(1-\exp(-\lambda d))$ bits. This bound is good for small λd.[2] For large λd Rohrs [3] showed that the amount of information decreases no faster than $1/(\lambda d)^2$, while strategies exist where it decreases like $\log(\lambda d)/(\lambda d)^2$ [2].

It should be stressed that these are bounds on the amount of information. This information must of course be transmitted as symbols. Quite unexpectedly at first sight the amount of ("physical") bits transmitted is known exactly, and does not depend on the actual strategy: $C(1-\rho)$ addressing bits are sent every second on a line of capacity C bits/s whose fraction ρ is occupied by actual message bits. It is so as all bits that do not belong to the bodies of the messages have an addressing function. This can be seen most clearly in the cyclic scan mode of addressing messages.

III. ALOHA-LIKE MULTIPLE ACCESS CHANNELS

III.1 Introduction

Consider the following model of the generalized ALOHA system. Geographically separated but time synchronized transmitters send and receive messages on a common channel. If no transmitter is active, this fact is recognized by all within t_o seconds. If exactly one transmitter sends a message, the message is received successfully and this is known to all within t_1 seconds. Finally, if two or more transmitters are active simultaneously, then a collision is said to occur and it is detected by all within t_2 seconds. All messages involved in the collision must be retransmitted at a later time.

This model represents a variety of systems. The slotted ALOHA channel [4] has $t_o = t_1 = t_2$. Carrier sense multiple access radio systems [5] can detect idles quickly (carrier not present) while they distinguish between collisions and successes by using error detecting codes. Thus they have $t_o << t_1 = t_2$. Some broadcast cable systems (e.g., the Ethernet [6]) have a "listen while transmit" feature that allows the quick abortion of transmission when a collision is detected. Thus typically $t_o = t_2 << t_1$. Finally "reservation" systems use short messages to reserve time for longer data messages. The short messages can be seen as an idle/collision detection mechanism, and again $t_o = t_2 << t_1$ [7].

The main challenge here is to find accessing schemes with small expected message delay, but in this paper we will focus on a slightly different problem and investigate how average message delay behaves when the number of sources grows large.

We define the <u>utilization</u> of a channel access scheme as the fraction of the time during which messages are successfully transmitted. We define the "<u>capacity</u>" of this channel as the supremum, over <u>all</u> schemes, of the utilization. If the number of transmitters is finite, then the capacity is 1. Simple schemes like synchronous time division multiplexing or round robin transmission (cyclic polling) avoid collisions and can achieve this capacity. Unfortunately they cause relatively long message delays when the generation rate of the messages is much smaller than $1/t_1$. In that case "random" transmission schemes are preferred. They allow collisions in the hope of reducing delay. Such random schemes are customarily analyzed assuming that there are infinitely many transmitters, each generating at most one message during its life-time, and that the global generation process of the messages is Poisson with rate λ.

The capacity of the channel under those conditions is still unknown. An early scheme, the slotted Aloha [4] strategy, has been said to have an utilization of 1/e (when $t_o = t_1 = t_2$), but has been shown to be unstable, i.e., with probability one its utilization decreases to 0 as time goes by. A new class of protocols has recently been proposed [8,9]. Each of those has a maximum utilization λ_o with the property that the number of messages which ave been generated but not yet successfully transmitted will be bounded with probability 1 as long as $\lambda t_1 < \lambda_o$.

If $\lambda t_1 \geq \lambda_o$, the utilization of the channel is λ_o, but the expected message delay is infinite. The largest λ_o found to this day is .4877 [10].

Pippenger first showed that the capacity is bounded away from 1, in fact is no more than .744 ($t_o=t_1=t_2$) [11]. He also generalized the model to include channels where the number of transmitted messages can be determined up to some maximum d, and has found a bound on the utilization that is strictly increasing with d, converging to 1. Moreover he showed the existence of strategies achieving utilization arbitrarily close to 1 when $d = \infty$. Humblet [12] generalized Pippenger's results for the use of different t_i's, and obtained a bound of .704 when d=2 and $t_o = t_1 = t_2$. This is bound is sketched below. All of the works just mentioned use information theoretic concepts. Recently Molle [13] obtained better results (.673 for $t_o = t_1 = t_2$) by an elegant and simple method. A slight generalization of his work is also presented.

Before proceeding with the derivations of the bounds we will examine the implication of these results. First, an algorithm that is efficient for infinitely many sources will also be efficient for $M < \infty$ sources as long as the typical intergeneration time at a source (M/λ for symmetric systems) is longer than the typical message delay. In that case each transmission at a source is independent of the previous one, and one might as well assume that all messages have distinct sources.

Secondly, the previously mentioned results show the existence of some number between .4877 and .673 such that if λt_1 is less than that number the average message delay can remain bounded no matter the value of M ($t_o = t_1 = t_2$). However, if $\lambda t_1 > C$, the average message delay must increase with M. It is readily seen that the increase is linear for synchronous time division multiplexing and cyclic polling.

III.2 The Information Theoretic Bound

To understand the following model, note that a conflict resolution protocol is a sequential decision process, thus it can be described as a ternary tree. Every node corresponds to an "experiment", i.e., the transmission of messages. Branches correspond to outcomes, i.e., idle, success, or collision. Associated with each experiment is a set of times, typically a time interval. Only those messages generated during the set corresponding to an experiment are transmitted when the experiment is made. Other conflict resolution algorithms rely on random choices, both ways are probabilistically equivalent when the generation times are Poisson.

A <u>protocol</u> for [0,T] is an infinite ternary tree in which there is an initial node called the root, and in which each node k is connected by branches to offsprings $k^{(i)}$, i = 0,1,2 that can be other nodes or leaves. Every node k is labelled with a measurable subset y(k) of [0,T]. $\mu(k)$ denotes the Lebesque measure of y(k) divided by T.

Let the random variables E denote a set of Poisson message generation times in [0,T], with expected cardinality ν. The <u>execution</u> of a protocol with respect to a finite set E in [0,T] is a path through the tree defined as follows. Let k_o, the first node on the path, be the root. Suppose that k_m has been determined, then $k_{m+1} = k_m^{(j)}$ where j is 0, 1 or 2, depending if the number of not yet transmitted messages in $E \cap y(k_m)$ is 0, 1 or more than 1.

The set of nodes k in an execution ℓ that have offsprings $k^{(1)}$ is denoted by S_ℓ, the set of sucessful experiments in ℓ.

A protocol will be called <u>valid</u> if, for almost every subset E of [0,T], the execution ℓ of the protocol with respect to E terminates after finitely many steps with $E \subset \bigcup_{k \in S_\ell} y(k)$, i.e., if every message has been successfully transmitted.

The set of nodes k in an execution ℓ that have offsprings $k^{(0)}$ or $k^{(1)}$ is denoted by T_ℓ, the set of experiments in ℓ not resulting in collisions.

A valid protocol will be called <u>minimal</u> if for all executions ℓ, $y(k) \cap y(k') = \phi$, $k \neq k'$, $k, \epsilon k'$ T_ℓ. Thus, in a minimal protocol, a subset of [0,T] is never tested again once it has been determined not to contain a message, or when the only message present has been successfully transmitted. Any valid protocol can be made minimal by iteratively changing the y(k)'s, starting from the root, so as to satisfy the null intersection property. The execution of the protocol with respect to a set E is not affected by the change.

The execution of a protocol is a random path through the tree. P(k) denotes the probability that node k is included in an execution, and q(k,i) denotes the conditional probability that $k^{(i)}$ follows k in the execution of a protocol.

The expected number of experiments, σ, in an execution of a protocol has value $\sigma = \sum_k P(k)$.

The expected fraction q_i of experiments resulting in outcome i is given by[1] (assuming $\sigma<\infty$)

$$q_i = \frac{1}{\sigma} \sum_k P(k)\, q(k,i) \qquad (1)$$

Note that $\sum_{i=0}^{2} q_i = 1$. For valid protocols

$$q_1 = \frac{\nu}{\sigma} \qquad (2)$$

We will denote (q_o, q_1, q_2) by $\underline{q}$.

The efficiency e of a protocol is simply

$e = \frac{\nu t_1}{\sigma \sum_{i=0}^{d} q_i t_i}$ where $t_i \geq 0$ is the time it takes to observe outcome i. Note that for valid protocols $e = \frac{\nu t_1}{\nu t_1 + \sigma \sum_{i \neq 1} q_i t_i} = \frac{t_1}{t_1 + f}$ where f is defined by $f = \frac{\sigma}{\nu} \sum_{i \neq 1} q_i t_i$ and can be thought of as the expected time overhead per message. Note that efficiencies close to 1 are achieved when $t_1 >> f$, which is typically the case for reservation and cable systems.

The previous relation between e and f allows us to lowerbound f (which does not depend on t_1) in order to upperbound e. We will show that $\underline{q}$ lies in some closed convex region S of the unit simplex. The minimum over that region of f considered as a function of $\underline{q}$ will be our lower bound.

We first note that for any execution ℓ of a minimal protocol $\sum_{k \in T_\ell} \mu(k) \leq 1$. Averaging over ℓ yields

$$\sum_k P(k)\mu(k)(q(k,o) + q(k,1)) \leq 1 \qquad (3)$$

Next the entropy h (i.e., minus the mean of the log of the probabilities) of the executions of a protocol can be written

$$h = \sum_k P(k)\, H(\underline{q}(k)) \qquad (4)$$

where $H(\underline{q}(k)) = - \sum_{i=0}^{2} q(k,i) \log q(k,i)$. The probability of an execution ℓ of a minimal protocol is more than $e^{-\nu} \prod_{b \in S_\ell} \nu\mu(b)$, as one arrival must have occured in every $y(b)$, $b \in S_\ell$, which are disjoint, and no arrival could have occurred outside such a subset. Thus, $h \geq E(-\log \prod_{k \in S_\ell} \nu\mu(k)e^{-\nu}) = \nu \log e - \sum_k P(k)q(k,1)\log(\nu\mu(k))$. The right hand side of the previous equality is not less that $\nu \log e - \sum_k P(k)q(k,1) \log \frac{q(k,1)}{q(k,0) + q(k,1)}$ as can be seen by using the inequality $\ln(x) \leq x-1$ and (3).

Subtracting this last expression from the right hand side of (4), dividing by σ and using (2) one obtains $\frac{1}{\sigma} \sum_k P(k)\, g(\underline{q}(k)) \geq 0$, where $g(\underline{x}) = - x_1 \log (x_o + x_1) - \sum_{\substack{i=0 \\ i \neq 1}}^{2} x_i \log(x_i) - x_1 \log e$. One can show that g is a strictly concave function, thus by Jensen's inequality and (1), $g(\underline{q}) \geq 0$.

To obtain a lowerbound on f, we find $\alpha = \min_{\underline{q} \in S} f = \min_{\underline{q} \in S} \frac{\sum_{i \neq 1} q_i t_i}{q_1}$ where $S = \{\underline{q} \in \mathbb{R}^3,\ q_i \geq 0,\ \sum q_i = 1,\ g(\underline{q}) \geq 0\}$.

It is straightforward [12] to find necessary and sufficient conditions for optimality and to numerically find the value of α. The results are given in Figure 1.

III.3 A Simpler Bound

The previous line of reasoning is interesting as it was the first to provide a bound on f, but another method, a slight generalization of Molle's genie argument [13], gives a better result while being much simpler.

Let us assume that we receive the help of a friendly genie which, every time a transmission results in a collision, pinpoints the two sources involved in the collision that were the first to generate their messages. The optimal policy under those conditions is obvious. We first specify a set of times and allow the transmissions of all the messages (expected value denoted by β) generated during those times. If the outcome is "idle" or "success" we just repeat the cycle. If the outcome is a collision we let the two souces pointed to by the genie transmit successively before repeating the cycle. It is not desirable to allow other sources to join in the transmissions, as the result would be equivalent to a probabilistic mixture of two deterministic policies: the one just outlined that guarantees success, and the one consisting of allowing everybody to transmit, which guarantees that the genie will point to the two oldest messages not yet transmitted. This last policy gives rise to a time overhead per message of at least $\frac{t_2}{2}$, which is not optimal as we shall see.

For a given β the expected time overhead per message is simply the ratio of the expected time overhead in a cycle to the expected number of messages successfully transmitted in a cycle.

Thus $f \geq \min_\beta \frac{t_o\, e^{-\beta} + t_2(1-e^{-\beta}-\beta\, e^{-\beta})}{\beta\, e^{-\beta} + 2\,(1-e^{-\beta}-\beta\, e^{-\beta})}$. This bound is also plotted in Figure 1. It is better than the previous bound for small values of t_o/t_2. For very small values of t_o/t_2 it is approximately equal to $\sqrt{2\, t_o t_2}$.

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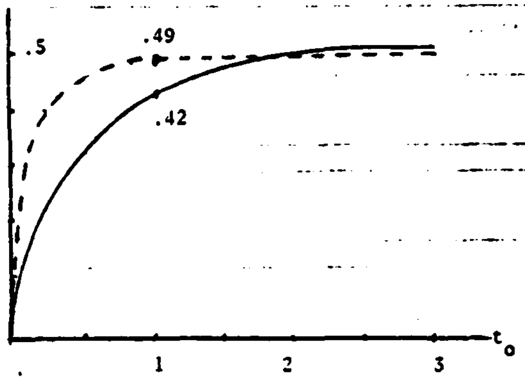

Figure I

Lower Bound on Time Overhead per Message $t_2 = 1$

——— Information Theoretic

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